In this excerpt, we lower the parameter *beta* in order to model a safe-sex program in fighting the spread of HIV.

In the case of COVID-19, the same parameter change can be used to model 'social distancing' = making the solution more dilute = increasing the mean free path. Since S and I stand for densities (like in chemistry), diluting the solution lowers the reaction rate, which is *beta*.

Epidemiology

In the study of disease transmission in populations, modeling has become an important practical tool. Early in this chapter we saw the results of the model that the CDC used to predict the course of the Ebola epidemic (Figure 1.8 on page 6). The type of model that the CDC used is called a "susceptible–infected" model, or sometimes an "SIR" model (where R stands for "recovered").

One of the early models used to study the epidemiology of HIV transmission was the model of Anderson and May (Anderson et al. 1992). We present here a slightly simplified version of their model (Figure 1.31).

We will assume three populations:

S Susceptible individuals, that is, people who are HIV negative

I Infected-but-not-yet-symptomatic individuals, who are HIV positive, and

A People with the symptoms of AIDS.

We assume a fixed population of 10,000 people. Assuming that the average life span is 75 years, we would expect 1/75 of the population to die each year, giving a person's probability of

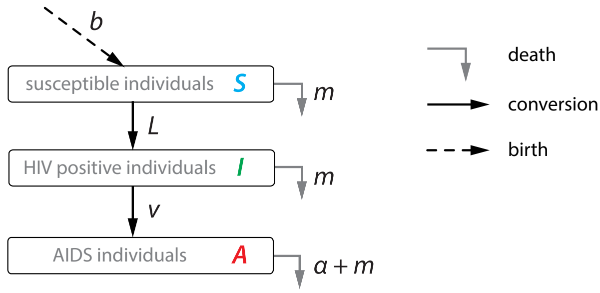

Figure 1.31: Schematic box-and-arrow diagram of a simplified version of the Anderson-May SI model for HIV transmission in a population.

dying is 1 in 75 years, giving a per capita death rate of $m = 1/(75 \text{ years})$. (Note that we are assuming that a person's probability of death is uniform across all ages, which is a limitation of this model. More advanced "age-structured" models use age-specific death rates.)

To compensate for these deaths, we also assume that $b = 133.3$ people are injected into the population each year, exactly making up for the natural death rate of $1/(75 \text{ years})$.

Exercise 1.4.23 Why 133.3? Where does this number come from?

The critical dynamical term is the susceptible-meets-infected term, which will have the form $S \times I$, just as it was with the sharks and tuna. Our underlying model here is a particularly simple one: we assume random encounters between members of S and I indiscriminately. In other words, we assume that neither party knows that an I is an I, that is, no one knows who is HIV+. Therefore, the probability that we encounter an I is $I/(S+I)$. We also assume that each person has, on average, c partners/year.

Of course, not every encounter between an S and an I results in the infection of the S. Just as in the shark–tuna model, there is a certain probability, which we call β, that the encounter will end in an infection. The parameter β is obviously extremely important: it is the parameter we can manipulate with safe sex practices and medications that reduce viral load and make infected people less likely to infect others. Let's begin by assuming that the probability of transmission of HIV with each encounter is a gloomy $\beta = 0.5$, or 50%.

Consequently, the overall per capita rate at which an S converts into an I is

$$L = c\beta \frac{I}{S+I}$$

We also need to reflect the fact that AIDS patients die more quickly than the average death rate. We assume an average AIDS-specific death rate of $\alpha = 1/(1 \text{ year})$. (This rate was more typical of the early days of the AIDS epidemic than it is today.) There is also a rate of conversion of I into A, that is, a rate of HIV+ people turning symptomatic, which we assume to take 8 years, giving a rate of conversion $I \to A$ of $1/(8 \text{ years})$ or 0.125/year.

The differential equations are therefore

$$\begin{aligned} S' &= b - (m+L)S \\ I' &= LS - (m+v)I \\ A' &= vI - (m+\alpha)A \end{aligned}$$

where

$$\begin{array}{llll} b = 133.33 & m = 1/75 & v = 0.125 & L = c\beta\dfrac{I}{S+I} \\ \alpha = 1 & c = 2 & \beta = 0.5 & \\ \text{initial conditions}: & S(0) = 9995 & I(0) = 5 & A(0) = 0 \end{array}$$

With these parameters, the model predicts that the populations will go to equilibrium values at approximately

$$S = 152 \qquad I = 949 \qquad A = 117$$

This is a very gloomy outcome: almost 9000 out of our original 10,000 have died after 20 years, with most of that coming in the first 10 years (Figure 1.32).

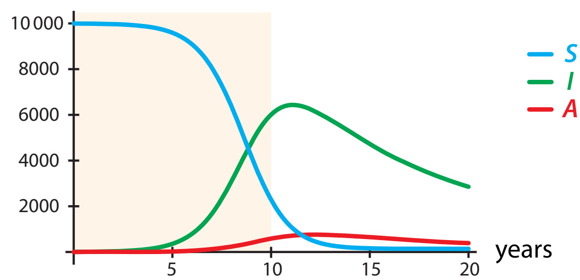

Figure 1.32: Time series output of the Anderson-May HIV model, assuming a high value for β. Note the outcomes.

But if we can change parameters, we can change outcomes. If we can lower β, for example by safe sex practices, to 0.05, the epidemic will die out. With this new value of β, a new equilibrium is reached, at approximately

$$S = 9994 \qquad I = 0 \qquad A = 0$$

indicating that we have prevented the virus from spreading (Figure 1.33).

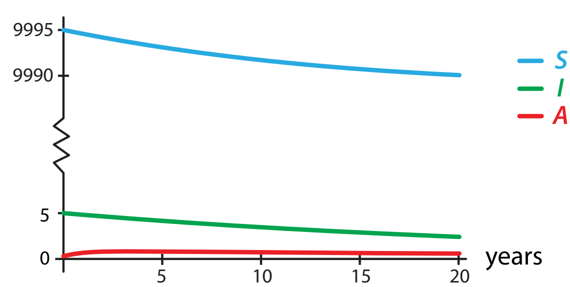

Figure 1.33: By lowering β, we can change the course of the epidemic.